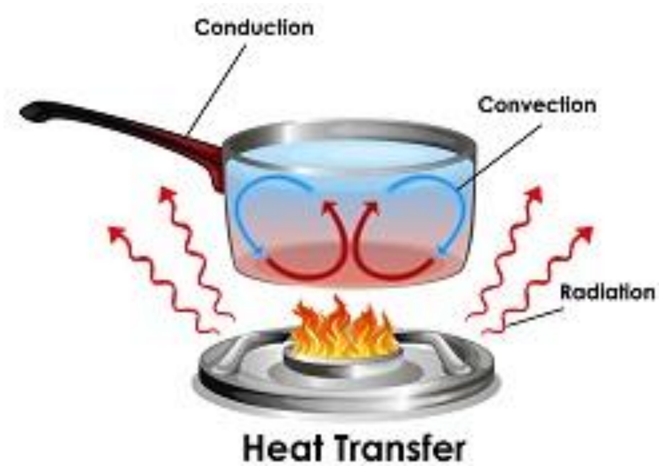
DEPARTMENT OF MECHANICAL ENGINEERING

ABIT

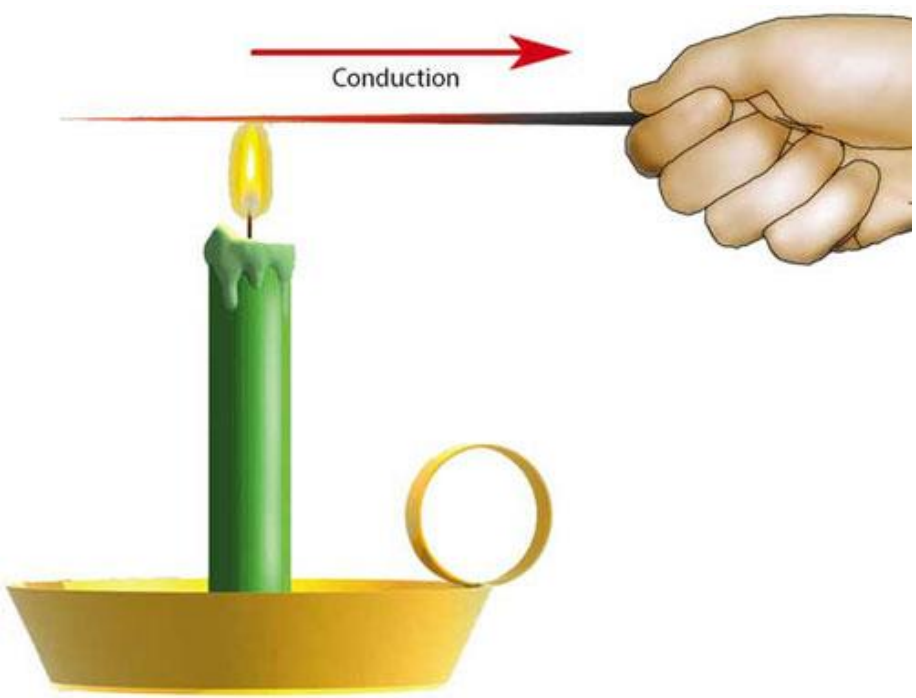
HEAT TRANSFER



NUMERICALS SOLVED

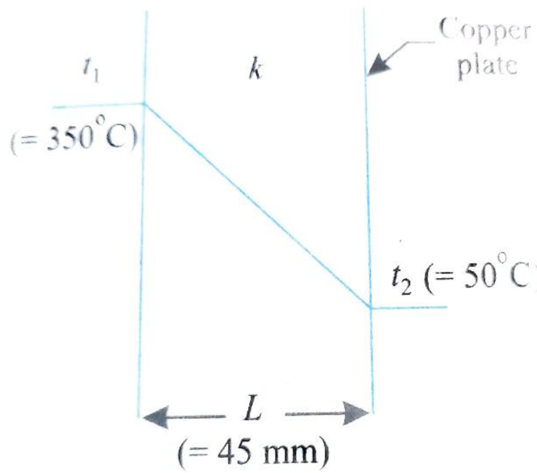
MODULE-I

CONDUCTION



Q. Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick , whose one face maintained at 350^{0}C and the other face at 50^{0}C.Take thermal conductivity of copper as 370W/m^{0}C.

Solution:



Temperature difference ,dt=t_2-t_1=50-350

Thickness of copper plate, L= 45mm =0.045m

Thermal conductivity of copper ,k =370W/m^{0}C

Rate of heat transfer per unit area ,q:

From Fourier's law

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

Or, $$q = \frac{Q}{A} = -k\,\frac{dt}{dx}$$

$$= -370X\,\frac{(50-350)}{0.045} = 2.466X10^6 W/m^2 = 2.466MW/m^2\,(Ans.)$$

Q. A wire 1.5mm in diameter and 150mm long is submerged in water at atmospheric pressure. An electric current is passed through the wire and is increased until the water boils at 100^0C. Under the condition if convective heat transfer coefficient is 4500W/m^2C find how much electric power must be supplied to the wire to maintain the wire surface t 120^0C?

Solution:

Diameter of the wire , d= 1.5mm = 0.0015 m

Length of the wire =L= 150mm =0.15 m

Surface area of the wire=A= πdL=π X 0.0015 X 0.15 =7.068X$10^{-4}$$m^2$

Wire surface temperature, t_s =120^0C

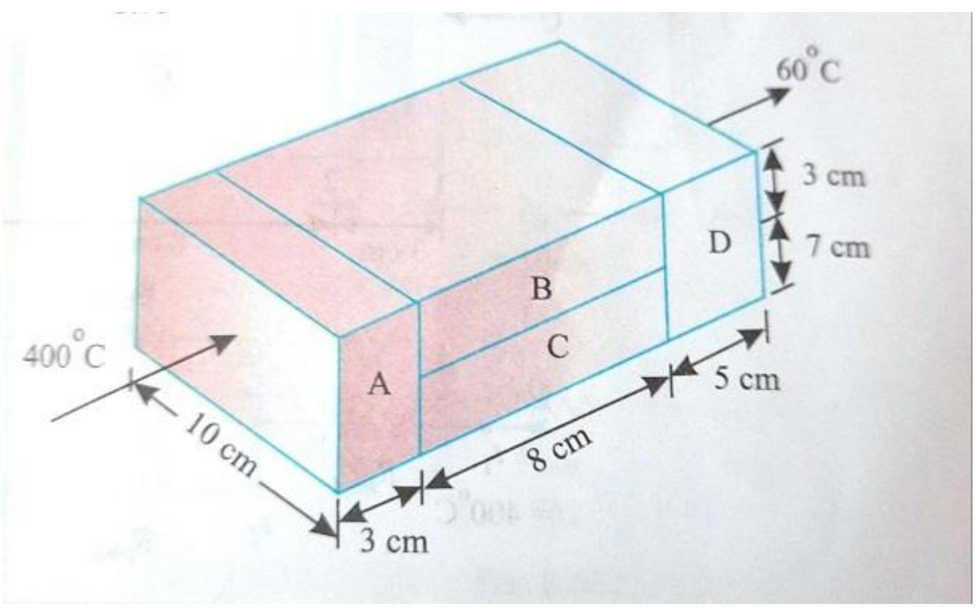
Water temperature, t_f =100^0C

Convective heat transfer coefficient, h=4500W/m^2C

Electric power to be supplied= Total convective loss(Q)

Q=hA(t_s-t_f)=4500 X 7.068 X 10^{-4} (120-100)=63.6 W (Ans.)

Q. Find the heat flow rate through the composite wall as shown in the figure. Assume one dimensional flow.



K_A =150 W/m^0C

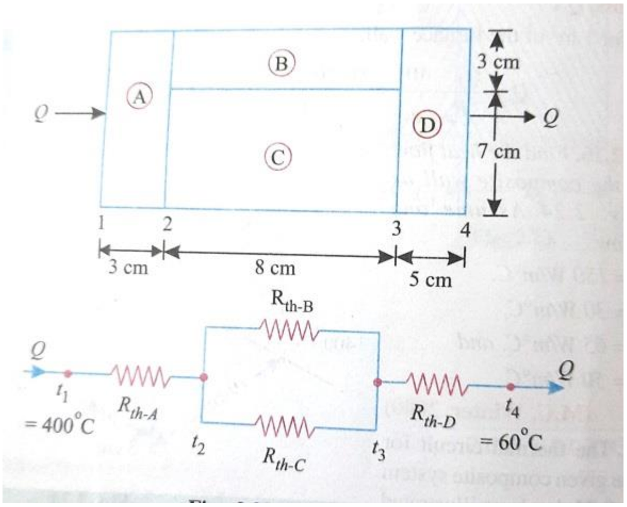
K_B =30 W/m^0C

K_C=65 W/m^{0}C

K_D =50 W/m^{0}C

Solution:

The thermal circuit for heat flow in the given composite system is shown in the figure below.



Thickness:

L_A =3 cm=0.03 m, L_B = L_C =8 cm =0.08 m, L_D =5 cm= 0.05 m

Areas:

A_A =0.1 X 0.1= 0.01 m^2, A_B =0.1 X 0.03 =0.003 m^2

A_C =0.1 X 0.07 =0.007 m^2, A_D =0.1 X 0.1 =0.01 m^2

Heat flow rate, Q:

The thermal resistances are given by,

$$R_{th-A} = \frac{L_A}{k_A A_A} = \frac{0.03}{150 X 0.01} = 0.02$$

$$R_{th-B} = \frac{L_B}{k_B A_B} = \frac{0.08}{30 X 0.003} = 0.89$$

$$R_{th-C} = \frac{L_C}{k_C A_C} = \frac{0.08}{65 X 0.007} = 0.176$$

$$R_{th-D} = \frac{L_D}{k_D A_D} = \frac{0.05}{50 X 0.01} = 0.1$$

The equivalent thermal resistance for the parallel thermal resistance R_{th-B} and R_{th-C} is given by

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176} = 6.805$$

$$(R_{th})_{total} = R_{th-A} + (R_{th})_{eq} + R_{th-D} = 0.02 + 0.147 + 0.1 = 0.267$$

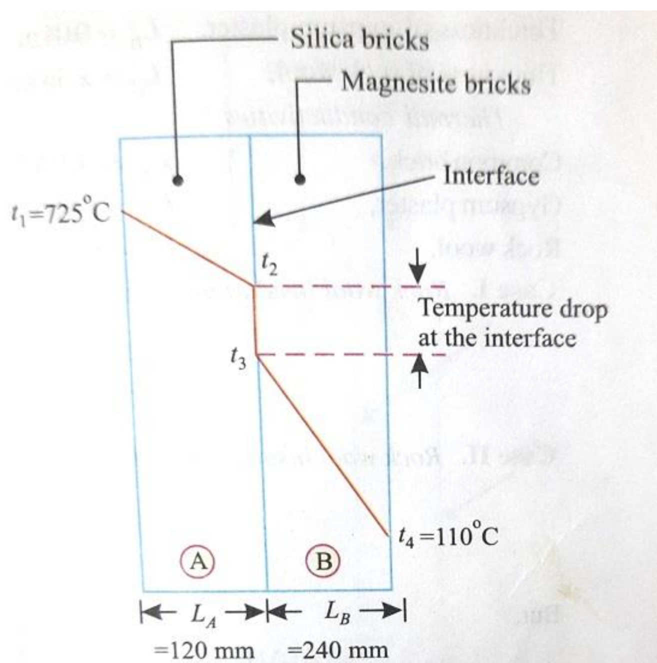
Hence, $$Q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(400-60)}{0.267} = 1273.4W(Ans.)$$

Q. A wall of a furnace is made up of inside layer of silica brick 120mm thick covered with a layer of magnesite brick 240 mm thick. The temperatures at the inside surface of silica brick wall and outside surface of magnesite brick wall are 725^0 C and 110^{0}C respectively. The contact thermal resistance between the two walls at the interface is 0.0035^{0}C/W per unit wall area. If thermal conductivities of silica and magnesite bricks are 1.7 W/m^{0}C and 5.8 W/m^{0}C, calculate,

(i) The rate of heat loss per unit area of walls, and

(ii) The temperature drop at the interface.

Solution:



Given: L_A =120 mm= 0.12 m,

L_B =240 mm = 0.24 m,

K_A = 1.7W/m^{0}C

K_B = 5.8 W/m^{0}C

The contact thermal resistance $(R_{th})_{cont.}$= 0.0035^{0}C/W

The temperature at the inside surface of silica brick wall, t_1 =725^{0}C

The temperature at the outside surface of the magnesite brick wall, t_4=110^{0}C

(i) The ratio of heat loss per unit area of wall ,q:

$$q = \frac{\Delta t}{\sum R_{th}} = \frac{\Delta t}{R_{th-A} + (R_{th})_{cont.} + R_{th-B}}$$

$$= \frac{(t_1 - t_4)}{\frac{L_A}{K_A} + 0.0035 + \frac{L_B}{k_B}} = \frac{(725 - 110)}{\frac{0.12}{1.7} + 0.0035 + \frac{0.24}{5.8}} = 5324.67 W/m^2$$

(ii) The temperature drop at the interface,$(t_2$-$t_3)$:

As the same heat flows through each layer of composite wall , therefore,

$$q = \frac{t_1 - t_2}{\frac{L_A}{k_A}} = \frac{t_3 - t_4}{\frac{L_B}{k_B}}$$

Or, $$5324.67 = \frac{(725 - t_2)}{\frac{0.12}{1.7}}$$

Or, $$t_2 = 349.14^0 C$$

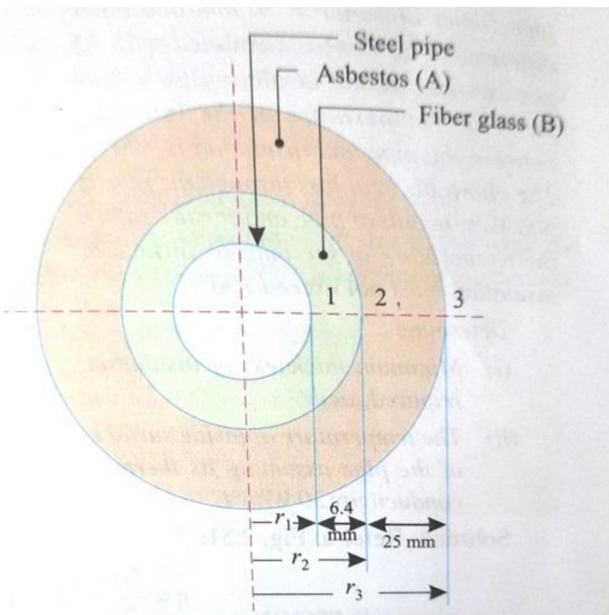
Similarly, $$5324.67 = \frac{(t_3 - 110)}{\frac{0.24}{5.8}}$$

Or, $$t_3 = 330.33^0 C$$

Hence the temperature drop at the interface=t_2-t_3=349.14-330.33=18.81^{0}C. (Ans.)

Q. A steel pipe with 50 mm OD is covered with a 6.4 mm asbestos insulation [k= 0.166 W/mK]followed by a 25 mm layer of fiber-glass insulation [k=0.0485W/mK] . The pipe wall temperature is 393 K and the outside insulation temperature is 311 K. Calculate the interface temperature between the asbestos and fiber-glass.

Solution:



Given: r_1 =50/2=25 mm=0.025 m:

r_2 -= r_1 +6.4=25+6.4=31.4 mm 0r 0.0314 m,

r_3 = r_2 + 25= 31.4+25= 56.4 mm= 0.0564 m,

T_1 =393 K, T_3 =311K

K_A =0.166 W/Mk

K_B =0.0485W/mK

Interface temperature between the asbestos and fiber-glass , t_2 :

We know that,
$$Q = \frac{2\pi L(T_1 - T_3)}{\frac{\ln\left(r_2/r_1\right)}{k_A} + \frac{\ln\left(r_3/r_2\right)}{k_B}}$$

$$\frac{Q}{L} = \frac{2\pi(T_1 - T_3)}{\frac{\ln\left(r_2/r_1\right)}{k_A} + \frac{\ln\left(r_3/r_2\right)}{k_B}}$$

$$= \frac{2\pi(393 - 311)}{\frac{\ln\left(0.0314/0.025\right)}{0.166} + \frac{\ln\left(0.0564/0.0314\right)}{0.0485}} = 38.31 W/m$$

Also ,
$$\frac{Q}{L} = \frac{2\pi(T_1 - T_2)}{\frac{\ln\left(r_2/r_1\right)}{k_A}}$$

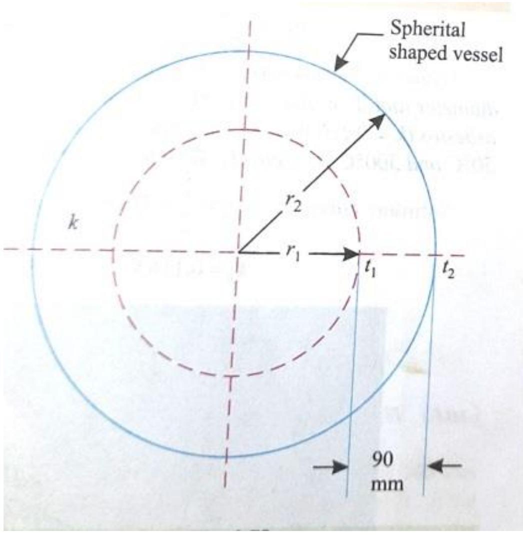
Or,
$$38.31 = \frac{2\pi(393 - T_2)}{\frac{\ln\left(0.0314/0.025\right)}{0.166}}$$

Or,
$$T_2 = 384.6K$$

Or, $t_2 = 384.6 - 273 = 111.6^0C$ (Ans.)

Q. A spherical shaped vessel of 1.4m diameter is 90 mm thick . Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 220^{0}C . Thermal conductivity of the material of the sphere is 0.083 W/m^{0}C.

Solution:



$r_2 = 1.4/2 = 0.7$ m

$r_1 = 0.7 - 0.090 = 0.61$ m

$t_1 - t_2 = 220^0C$

k = 0.083 W/m^{0}C

The rate of heat leakage is given by

$$Q = \frac{(t_1 - t_2)}{\left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2}\right]} = \frac{220}{\left[\frac{(0.7 - 0.61)}{4\pi X\, 0.083 X\, 0.61 X\, 0.7}\right]} = 1088.67W\,(Ans)$$

Q. Calculate the critical radius of insulation for asbestos [k=0.172 W/mK] surrounding a pipe and exposed to room air at 300 K with h=2.8 W/mK . Calculate the heat loss from a 475 K, 60 mm diameter pipe when covered with the critical radius of insulation and without insulation.

Solution:

Given: k= 0.172 W/mK, T_1 =475 K, T_2 = 300 K,

h_o =2.8 W/m^{2}K, r_1=60/2=30 mm=0.03 m.

The critical radius of insulation,

$$r_c = \frac{k}{h_o} = \frac{0.172}{2.8} = 0.06143 m or 61.43 mm (Ans.)$$

Q (with insulation) $= \dfrac{2\pi(T_1 - T_2)}{\dfrac{\ln\left(r_c/r_1\right)}{k} + \dfrac{1}{h_o r_c}} = \dfrac{2\pi(475-300)}{\dfrac{\ln\left(0.06143/0.03\right)}{0.172} + \dfrac{1}{2.8X\,0.06143}} = 110.16W/m(Ans.)$

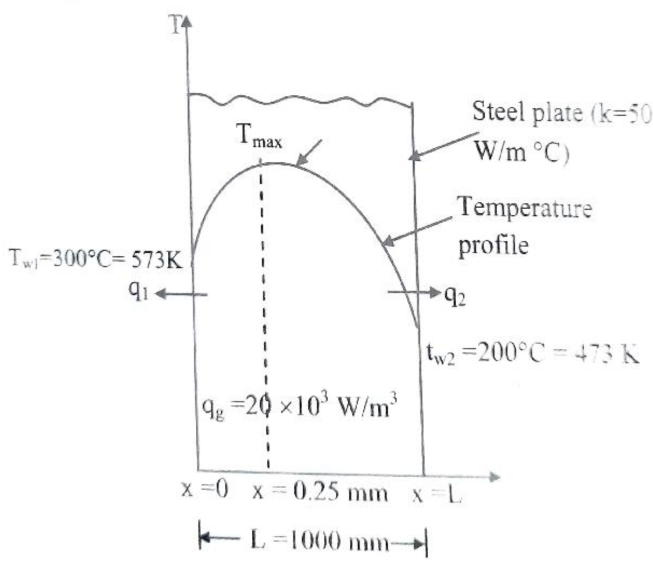
Q (without insulation) $= h_o X\,2\pi r_1(T_1 - T_2) = 2.8X\,2\pi X\,0.03(475-300) = 92.36W/m(Ans.)$

Q. The temperatures on the two surfaces of a 1000 mm thick steel plate , (k= 50 W/m^{0}C) having a uniform volumetric heat generation of 20X 10^3 W/m^3, are 300^{0}C and 200^{0}C . Determine the following :

(i) the temperature distribution across the plate:

(ii) the value and position of the maximum temperature.

Solution:



Given,

L= 1000 mm=1m

T_{W1} =573 K, T_{W2} =473 K

q_g=20X10^3 W/m^3,

k=50W/m^{0}C

(i) The temperature distribution , when both the surfaces of the wall have different temperatures is given by

$$T = T_{W1} + \left(\frac{q_g}{2k}(L-x) + \frac{(T_{W2} - T_{W1})}{L} \right) x$$

Substituting the given values

$$T = 573 + \left(\frac{20000}{2X50}(1-x) + \frac{(473-573)}{1} \right) x = 573 + (200(1-x) - 100)x$$

$$T = 573 + 100x - 200x^2$$

(ii) In order to determine the position of maximum temperature, differentiating the above expression and equating it to zero,we get

$$\frac{dT}{dx} = 100 - 400x = 0$$

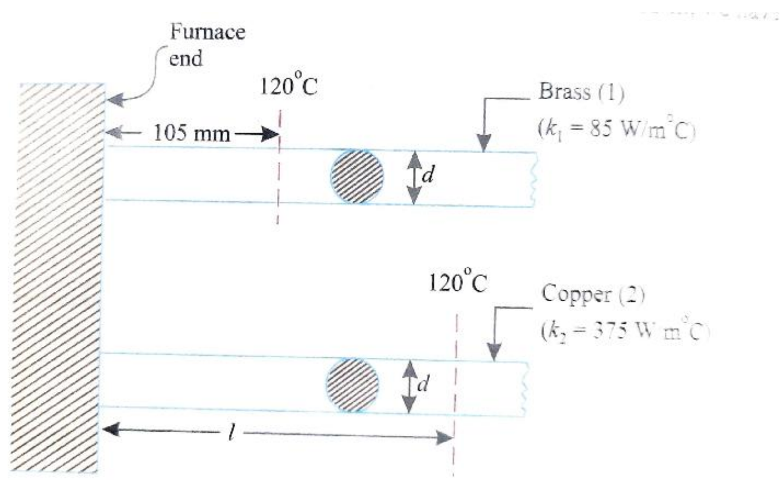
Or, x=0.25 m 0r 250 mm

The maximum temperature is

$$T_{\max} = 573 + 100X\,0.25 - 200X\,0.25^2 = 585.5K = 312.5^0C$$

Q. Two long rods of the same diameter, one made of brass (k=85W/m^{0}C) and other made of copper (k= 375W/m^{0}C) have one of their ends inserted into the furnace. Both of the rods are exposed to the same environment. At a distance 105 mm away from the furnace end, the temperature of the brass rod is 120^{o}C. At what distance from the furnace end the same temperature would be reached in the copper rod?

Solution:



K_1 =85W/m^{0}C, k_2= 375 W/m^{0}C

The controlling differential equation for the heat flow in the rod is given by

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

The general solution is

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \ldots\ldots\ldots\ldots\ldots(1)$$

The boundary conditions are:

(i) At x=0, $\theta=\theta_0$

(ii) At x= ∞ , $\theta=0$

Using boundary condition (ii) , we get

$$0 = C_1 e^{mx} \Rightarrow C_1 = 0$$

$$\theta = C_2 e^{-mx} \ldots\ldots\ldots\ldots\ldots(2)$$

Now using boundary condition (i), we get

$$\theta_0 = C_2 \Rightarrow \theta = \theta_0 e^{-mx}$$

Or, $$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = e^{-mx} \Rightarrow t = t_a + (t_0 - t_a)e^{-mx} \ldots\ldots\ldots\ldots\ldots(3)$$

Now usng equation(3) for brass rod, when t=120^{0}C at x=105 mm=0.105 m, we have

$$120 = t_a + (t_0 - t_a)e^{(-m_l X\, 0.105)} \ldots\ldots\ldots\ldots(4)$$

Now using equation(3) for copper rod, when t=120^{0}C at x=l, we have

$$120 = t_a + (t_0 - t_a)e^{-m_2 l} \ldots\ldots\ldots\ldots(5)$$

As t_0 and t_a are same for both the rods.

Equating (4) and (5) , we get

$$e^{-m_1 X\, 0.105} = e^{-m_2 l} \Rightarrow 0.105 m_1 = m_2 l$$

Or, $$l = \frac{m_1}{m_2} X\, 0.105$$

But, $m_1 = \sqrt{\dfrac{hP}{k_1 A_{cs}}} and m_2 = \sqrt{\dfrac{hP}{k_2 A_{cs}}}$

And, $\dfrac{m_1}{m_2} = \sqrt{\dfrac{k_2}{k_1}} = \sqrt{\dfrac{375}{85}} = 2.1$

Hence, l=2.1 X 0.105=0.22 m or 220 mm(Ans.)

Q. A fin 5 mm thick and 40 mm long has its base on a plane plate which is maintained at 150^0C. The ambient temperature is 30^0C. The conductivity of fin material is 50 W/m^{0}C and the heat transfer coefficient is 200 W/m^{20}C. Determine temperatures at the end of the fin?

Solution: L=0.04 m, b=1m, y=5 mm=0.005 m,

K= 50 W/m^{0}C, h=200 W/m^{20}C,

T_b =150^{0}C, T_a=30^{0}C

(i) Temperature at the end of the fin, T_l:

$$\frac{T - T_a}{T_b - T_a} = \frac{\cosh m(l-x) + \dfrac{h}{mk}\sinh\ m(l-x)}{\cosh(ml) + \dfrac{h}{mk}\sinh(ml)}$$

At x=l

$$\frac{T_l - T_a}{T_b - T_a} = \frac{1}{\cosh(ml) + \dfrac{h}{mk}\sinh(ml)}$$

Where, $m = \sqrt{\dfrac{hP}{kA_{cs}}} = \sqrt{\dfrac{hX(2b+2y)}{kX(bXy)}}$

Since 2y<<2b

$$\sqrt{\frac{h}{k} X \frac{2}{y}} = \sqrt{\frac{200}{50} X \frac{2}{0.005}} = 40$$

Ml=40X0.04=1.6

Hence, $\dfrac{T_l - 30}{150-30} = \dfrac{1}{\cosh(1.6) + \dfrac{200}{50X40}\sinh(1.6)} \Rightarrow T_l = 72.6^0 C(Ans.)$

Q. Find out the amount of heat transferred through a fin whose cross-sectional area is 0.005m^2 and surface area is 0.5m^2 and perimeter is 0.2m^2. The temperature at the base of the fin is 100^{0}C Also determine the temperature at tip of the fin, if the atmosphere temperature is 30^{0}C. Assume fin length=5cm, k=20W/m^{0}C and h=50 W/m^{20}C and end of the fin is insulated.

Solution:

Given, surface area A_{su}=0.5m^2

Cross-sectional area A_{cs}=0.005m^2

Perimeter P=0.2m^2

K=20W/m^{0}C, h=50 W/m^{20}C

T_o=100^{0}C, T_a=30^{0}C

Amount of heat transferred through the fin, Q:

$$Q = \sqrt{PhkA_{cs}}\,(T_o - T_a)\tanh(ml)$$

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{50X0.2}{20X0.005}} = 10$$

$$Q = \sqrt{0.2X50X20X0.005}X(100-30)\tanh(10X0.05) = 32.35W$$

Temperature at the tip of the fin, Qx=L:

$$\frac{\theta}{\theta_0} = \frac{T - T_a}{T_o - T_a} = \frac{\cosh(m(l-x))}{\cosh(ml)}$$

At x=l, we have

$$\frac{T-30}{100-30} = \frac{1}{\cosh ml} \Rightarrow T = 92.07^0 C(Ans.)$$

Q. A longitudinal copper fin (k=380 W/m^{0}C) 600 mm long and 5 mm diameter is exposed to air stream at 20^{0}C. The convective heat transfer coefficient is 20W/m^{20}C. If the fin base temperature is 150^{0}C , determine:

(i) The heat transferred, and

(ii) The efficiency of the fin.

Assume fin is insulated at the tip.

Solution:

Length of the fin , l=600 mm =0.6 m

Diameter of the fin, d= 5 mm =0.005 m

The fin base temperature , t_0 =150^{0}C

Air stream temperature, t_a =20^{0}C

Thermal conductivity of fin material, k= 380 W/m^{0}C

Convective heat transfer coefficient h =20 W/m^{20}C

(i)The heat transferred, Q :

$$Q = kA_{cs} m(t_0 - t_a)\tanh(ml)$$

Where, $m = \sqrt{\dfrac{hP}{kA_{cs}}} = \sqrt{\dfrac{h}{2} x \dfrac{\pi d}{\dfrac{\pi}{4} d^2}} = \sqrt{\dfrac{4h}{kd}} = \sqrt{\dfrac{4X20}{380X0.005}} = 6.49$

Hence, $Q = 380X\left(\dfrac{\pi}{4}X0.005^2\right)X6.49X(150-20)\tanh(6.49X0.6) = 6.29W$

(ii) The efficiency of the fin η_{fin} :

$$\eta_{fin} = \frac{\tanh(ml)}{ml} = \frac{\tanh(6.49X0.6)}{(6.49X0.6)} = 0.2566 = 25.66\%(Ans.)$$

Q. A solid copper sphere of 10 cm diameter[ρ=8954 kg/m³ , c_p = 383 J/kg K , k= 386 W/mK],initially at a uniform temperature t_i =250^{0}C, is suddenly immersed in a well stirred fluid which is maintained at a uniform temperature t_a =50^{0}C. The heat transfer coefficient between the sphere and the fluid is h=200 W/m²K. Determine the temperature of the copper block at τ= 5 min after the immersion.

Soluton:

Given: D=10 cm = 0.1 m,ρ=8954 kg/m^3, c_p =383 J/kgK, k= 386 W/mK , t_i =250^{0}C,t_a =50^{0}C, h=200W/m^{2}K, τ=5 min =300 s.

Temperature of the copper block, t:

The characteristic length of the sphere is

$$L_c = \frac{volume(V)}{surfacearea(A_s)} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6} = \frac{0.1}{6} = 0.0167m$$

Biot number, $$Bi = \frac{hL_c}{k} = \frac{200X0.0167}{386} = 8.64X10^{-3}$$

Since Bi is less than 0.1, hence lump capacitance method may be applied for the solution of the problem.

The temperature distribution is given by

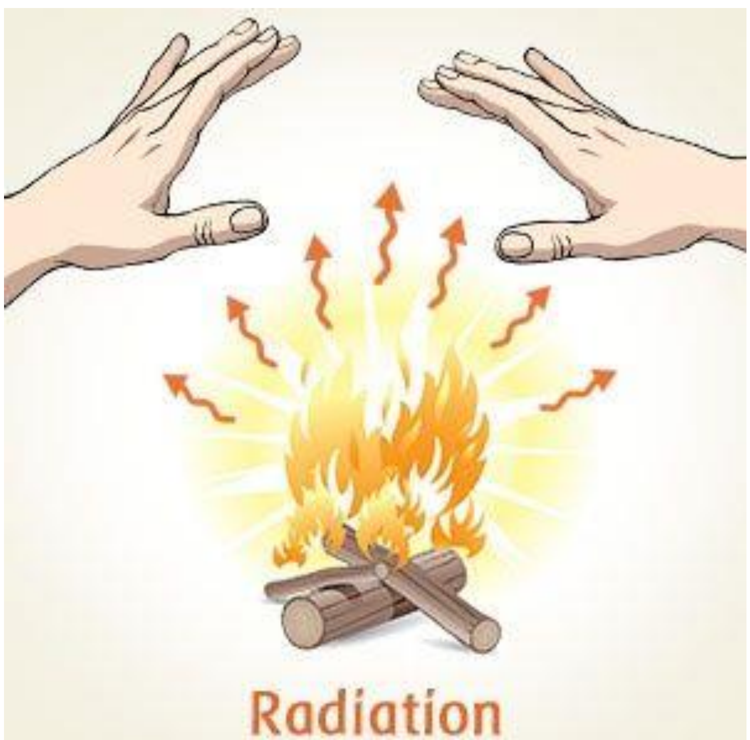
$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}.\tau\right] \Rightarrow \frac{t-50}{250-50} = \exp\left[-\frac{200}{8954X0.0167X383}X300\right] = 0.35$$

Since $\frac{A_s}{V} = \frac{L}{L_c} = \frac{1}{0.0167}$

Hence, t=(250-50)X0.35+50=120^{0}C (Ans.)

MODULE-III

RADIATION



Q. Incident radiation (1577W/m²) strikes an object. The amount of energy absorbed is 472 W/m² and amount of energy transmitted is 78.8 W/m². What is the value of the reflectivity?

Solution:

Energy balance

$Q_i = Q_r + Q_\tau + Q_a$

Where , Q_i =incident radiation

Q_r = reflected radiation

Q_a = absorbed radiation

Q_τ = transmitted radiation

$$\Rightarrow 1 = \frac{Q_r}{Q_i} + \frac{Q_\tau}{Q_i} + \frac{Q_a}{Q_i} \Rightarrow 1 = \frac{Q_r}{Q_i} + \frac{78.8}{1577} + \frac{472}{1577} \Rightarrow 1 = \rho + 0.05 + 0.3 \Rightarrow \rho = 0.65$$

Where,ρ is the reflectivity. (Ans.)

Q. A black body 0f 0.5m² area has an effective temperature of 527⁰C. Calculate the intensity of radiation along a direction at 60⁰ to the normal.

Solution:

Intensity of normal radiation(I_n)= $\dfrac{\sigma T^4}{\pi} = \dfrac{5.67 X 10^{-8} X (527+273)^4}{\pi} W/m^2 = 7392.53 W/m^2$

From Lambert's cosine law

$$I_\phi = I_n . \cos\phi = 7392.53 X \cos 60^0 = 3696.26 W/m^2 (Ans.)$$

Q. Assuming the sun to be a black body emitting radiation with maximum intensity at λ =0.49µm, calculate the following:

(i) The surface temperature of the sun, and

(ii) The heat flux at surface of the sun.

Solution:

Given, λ_{max} =0.49µm

(i) According to Wien's displacement law

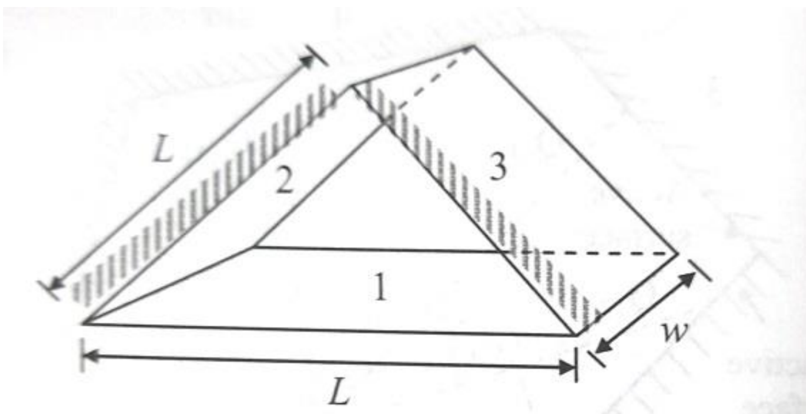
$\lambda_{max} T$=2898 µmK

$T=2898/\lambda_{max}=2898/0.49=5914$ K.

Heat flux

$$=(E)_{sun}=\sigma T^4=5.67X10^{-8}T^4=5.67\left(\frac{T}{100}\right)^4=5.67X\left(\frac{5914}{100}\right)^4=6.936X10^7 W/m^2\,(Ans.)$$

Q. Determine all possible shape factor for equilateral duct shown in the figure.



Solution:

Number of shape factors for N body enclosures= $N^2=3^2=9$

Plane wall 1,2,3

$F_{11}=F_{22}=F_{33}=0$

Surface 1:

$F_{11}+F_{12}+F_{13}=1$

$F_{12}=F_{13}$ (Symmetry)

$F_{12}+F_{13}=1$

$F_{12}=F_{13}=0.5$

$A_1F_{12}=A_2F_{21}$

If $A_1=A_2$

$F_{21}=F_{12}=0.5$

$A_1F_{13}=A_3F_{31}$

If $A_1=A_3$

$F_{13}=F_{31}=0.5$

$F_{22}+F_{21}+F_{23}=1$

$F_{22}=0$

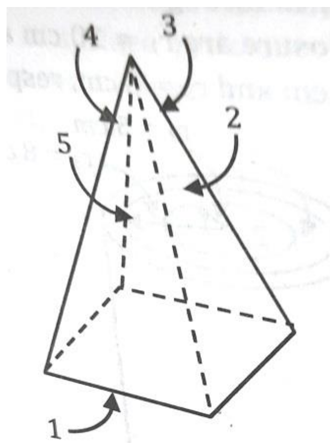
$F_{21}=F_{23}=0.5$(Symmetry)

$F_{31}+F_{32}+F_{33}=1$

$F_{33}=0$

$F_{31}=F_{32}=0.5$(Symmetry)

Q. Determine the view factors from the base of the pyramid shown in the figure to each of its four side surfaces. The base of the pyramid is a square and its side surfaces are isosceles triangles.



The Pyramid

Solution:

The base of the pyramid (surface 1) and its four side surfaces (surfaces 2, 3 ,4 and 5) form a five surface enclosure.

The four side surfaces are symmetric about the base surface.

Then , by symmetry rule, we have

$F_{12}=F_{13}=F_{14}=F_{15}$

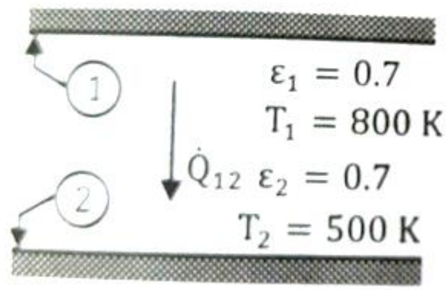
Also , by summation rule

$F_{11}+F_{12}+F_{13}+F_{14}+F_{15}=1$

But $F_{11}=0$, since base is a flat surface.

Hence $F_{12}=F_{13}=F_{14}=F_{15}=0.25$ (Ans.)

Q. Two very large parallel plates are maintained at uniform temperatures T_1=800K and T_2=500K and have emissivities ϵ_1=0.2 and ϵ_2=0.7, respectively as shown in the figure. Determine the net rate of radiation heat transfer between the two surfaces per unit surface area of the plates.



Solution:

$$Q=\frac{Q_{12}}{A}=\frac{\sigma(T_1^4-T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}=\frac{\left(5.67X10^{-8}\right)\left[(800)^4-(500)^4\right]}{\frac{1}{0.2}+\frac{1}{0.7}-1}=3625W/m^2\,(Ans.)$$

Q. A gray, diffuse opaque surface (α=0.8) is at 100^0C and receives irradiation of 1000 W/m^2. If the surface area is 0.1m^2.

Determine

- **(i) Radiosity of the surface and net radiation heat transfer from the surface.**
- **(ii) Calculate Radiosity and net radiation heat transfer if the surface is black.**

Solution:

Given- for gray surface α=ϵ=0.8, T=100^0C=373K, G=1000W/m^2, A=0.1m^2

(i) For opaque surface τ=0

α+ρ=1

ρ=1-0.8=0.2

Radiosity

$$J=\varepsilon E_b+\rho G=0.8\sigma T^4+0.2X1000=0.8X5.67X10^{-8}X373^4+0.2X1000=1078W/m^2$$

The net heat transfer rate

$$Q = A(J - G) = 0.1(1078 - 1000) = 7.8W/m^2$$

(ii) For black surface α=ϵ=1

$$J = E_b = \sigma T^4 = 5.67X10^{-8} X 373^4 = 1097.5W/m^2$$

$$Q = A(J - G) = A(E_b - G) = 0.10(1097.5 - 1000) = 9.75W/m^2$$

Q. Two large parallel planes having emissivities at 0.3 and 0.5 are maintained at temperature of 800^{0}C and 300^{0}C respectively. A radiation shield having an emissivity of 0.05. Calculate

(i) Heat transfer per unit area without shield

(ii) The temperature of the shield

(iii) Heat transfer per unit area with shield

Solution:

(i) Without shield heat transfer is given by

$$\frac{Q}{A} = q = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{5.67X10^{-8}(1073^4 - 573^4)}{\frac{1}{0.3} + \frac{1}{0.5} - 1} = 15934W/m^2 = 15.934kW/m^2$$

(ii) Let temperature of the shield=T_3

For steady state condition

$$q = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \Rightarrow \frac{1073^4 - T_3^4}{\frac{1}{0.3} + \frac{1}{0.05} - 1} = \frac{T_3^4 - 573^4}{\frac{1}{0.05} + \frac{1}{0.5} - 1} \Rightarrow T_3 = 875.2K$$

Hence, temperature of this shield =875.2-273=602.2^{0}C

(iii) Heat lost with shield

$$q = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{5.67X10^{-8}(1073^4 - 875.2^4)}{\frac{1}{0.3} + \frac{1}{0.05} - 1} = 1875.78W/m^2 = 1.875kW/m^2$$

Q. Determine the heat lost by radiation per meter length of 80 mm diameter pipe at 300^{0}C if

(i) Located in a large room with red brick walls at a temperature of 27^{0}C.

(ii) Enclosed in a 16 cm diameter red brick conduit at a temperature of 27^{0}C.

For steel pipe ϵ =0.79

For brick conduit ϵ =0.93

Solution:

(i) Heat transfer by radiation

$$Q = \sigma A\varepsilon(T_1^4 - T_2^4) = 5.67X10^{-8}(\pi X0.08X1)0.79(573^4 - 300^4) = 1122.4W/m$$

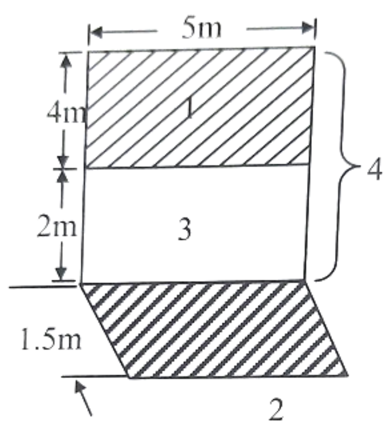
(ii) Heat transfer by radiation with conduit

$$Q_{1-2} = \frac{\sigma A(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

Where $\frac{A_1}{A_2} = \frac{\pi D_1}{\pi D_2} = \frac{8}{16} = \frac{1}{2}$

$$Q_{1-2} = \frac{5.67X10^{-8}(\pi X0.08X1)(573^4 - 300^4)}{\frac{1}{0.79} + \frac{1}{2}\left(\frac{1}{0.93} - 1\right)} = 1089.99W/m$$

Q. Find the shape factor for the given configuration and heat exchange between 1 and 2 if T_1=400^{0}C, T_2=200^{0}C, ϵ_1=0.8 and ϵ_2=0.9. Use F_{2-4} =0.35, F_{2-3} =0.29.



Solution:

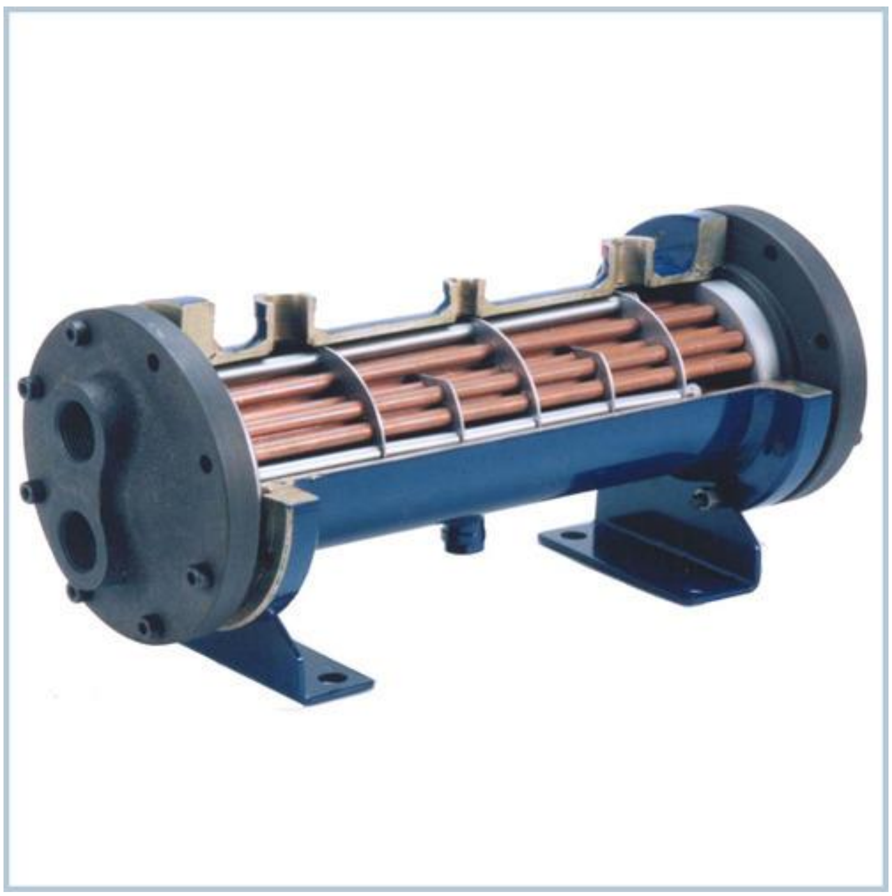
From reciprocity theorem $A_1F_{1-2}=A_2F_{2-1}$

$$F_{1-2} = \frac{A_2}{A_1}(F_{2-1}) = \frac{A_2}{A_1}(F_{2-4} - F_{2-3}) = \frac{1.5X5}{4X5}(0.35 - 0.29) = 0.0225$$

$$Q = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\left(\frac{A_1}{A_2}\right)} = \frac{5.67X10^{-8}X20(673^4 - 473^4)}{\left(\frac{1-0.8}{0.8}\right) + \frac{1}{0.0225} + \left(\frac{1-0.9}{0.9}\right)X\left(\frac{20}{7.5}\right)} = 3909W$$

MODULE-IV

HEAT EXCHANGER



Q. In a counter-flow double pipe heat exchanger, water is heated from 25^0C to 65^0C by an oil with a specific heat of 1.45 Kj/kgK and mass flow rate of 0.9 kg/s. The oil is cooled from 230^0C to 160^0C. If the overall heat transfer coefficient is 420 W/m^{20}C, calculate the following:

i. **The rate of heat transfer,**

ii. **The mass flow rate of water, and**

iii. **The surface area of the heat exchanger.**

Solution:

Given:

t_{c1} =25^0C, t_{c2} =65^0C, c_{ph}=1.45kJ/kgK, m_h =0.9kg/s,

t_{h1} =230^0C, t_{h2} =160^0C ,U=420 W/m^{20}C

(i) The rate of heat transfer,Q:

$$Q = \dot{m}_h \; Xc_{ph} X(t_{h1} - t_{h2}) = 0.9X1.45X(230-160) = 91.35kJ/s(Ans.)$$

(ii) The mass flow rate of water,m_c:

Heat lost by oil(hot fluid)=heat gained by water(cold fluid)

$$\dot{m}_h \; Xc_{ph} X(t_{h1} - t_{h2}) = \dot{m}_c \; Xc_{pc} X(t_{c2} - t_{c1}) \Rightarrow 91.35 = \dot{m}_c \; X4.187X(65-25)$$

$$\dot{m}_c = \frac{91.35}{4.187X(65-25)} = 0.545kg/s(Ans.)$$

(iii) The surface area of heat exchanger, A:

LMTD is given by

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\theta_1 / \theta_2\right)} = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\ln[(t_{h1} - t_{c2})/(t_{h2} - t_{c1})]} = \frac{(230-65)-(160-25)}{\ln[(230-65)/(160-25)]}$$

$$\theta_m = 149.5^0 C$$

Also, $Q = UA\theta_m \Rightarrow A = \dfrac{Q}{U\theta_m} = \dfrac{91.35X10^3}{420X149.5} = 1.45m^2(Ans.)$

Q. A hot fluid at 200^{0}C enters a heat exchanger at a mass flow rate of 10^4 kg/h. Its specific heat is 2000J/kgK .It is to be cooled by another fluid entering at 25^{0}C with a mass flow rate 2500kg/h and specific heat 400 J/kgK. The overall heat transfer coefficient based on outside area of 20 m^2 is 250 W/m^{2}K. Find the exit temperature of the hot fluid when the fluids are in parallel flow.

Solution:

Given: t_{h1} =200^{0}C , m_h =10000/3600=2.78 kg/s , c_{ph} = 2000 J/kgK , t_{c1} =25^{0}C

M_c = 2500/3600=0.694 kg/s , c_{pc} =400 J/kgK ,U= 250W/m^{2}K.

Exit temperature of the hot fluid, t_{h2}:

Heat lost by the hot fluid,

$$Q = \dot{m}_h c_{ph}(t_{h1} - t_{h2}) = 2.78 X\, 2000 X\,(200 - t_{h2}) = 5560(200 - t_{h2}) ....(1)$$

Heat gained by the cold fluid, $Q = 0.694 X\, 400(t_{c2} - 25) = 277.6(t_{c2} - 25) ....(2)$

Equating (1) and (2),

$$5560(200 - t_{h2}) = 277.6(t_{c2} - 25)$$

Or, $t_{c2} = 4025 - 20 t_{h2} ....(3)$

Als0, $Q = UA\theta_m$

Where, $\theta_m = \dfrac{\theta_1 - \theta_2}{\ln(\theta_1 / \theta_2)}$

Here, $\theta_1 = t_{h1} - t_{c1} = 200 - 25 = 175^0 C and \theta_2 = t_{h2} - t_{c2}$

$$\theta_m = \frac{175 - (t_{h2} - t_{c2})}{\ln\left[\dfrac{175}{t_{h2} - t_{c2}}\right]}$$

Substituting the values in the above equation, we get

$$Q = 250 X\, 20 \left[\frac{175 - (t_{h2} - t_{c2})}{\ln\left[\dfrac{175}{t_{h2} - t_{c2}}\right]}\right] ....(4)$$

Substituting the values of t_{c2} from (3) in (4), we have

$$Q = 250X\,20\left[\frac{175-\{t_{h2}-(4025-20t_{h2})\}}{\ln\left\{\frac{175}{t_{h2}-(4025-20t_{h2}}\right\}}\right] = 5000\left[\frac{175-(21t_{h2}-4025}{\ln\left\{\frac{175}{21t_{h2}-4025}\right\}}\right] \text{..................}(5)$$

Equating (1) and (5), we have

$$5560(200-t_{h2}) = 5000\left[\frac{175-(21t_{h2}-4025}{\ln\left\{\frac{175}{21t_{h2}-4025}\right\}}\right]$$

By hit and trial method, the value of t_{h2} may be found out. (Ans.)

Q. In a parallel flow heat exchanger water flows through the inner pipe and is heated from 20^{0}C to 70^{0}C. Oil flowing through the annulus is cooled from 200^{0}C to 100^{0}C. It is desired to cool the oil to a lower exit temperature by increasing the length of the heat exchanger. Determine the minimum temperature to which the oil may be cooled.

Solution:

Let , t_{h1} =200^{0}C , t_{h2} =100^{0}C

t_{c1} =20^{0}C , t_{c2} =70^{0}C

$$Q = \dot{m}_h\, c_{ph}(t_{h1}-t_{h2}) = \dot{m}_c\, c_{pc}(t_{c2}-t_{c1}) \Rightarrow \dot{m}_h\, c_{ph}(200-100) = \dot{m}_c\, c_{pc}(70-20)$$

Or, $$\frac{\dot{m}_c\, c_{pc}}{\dot{m}_h\, c_{ph}} = \frac{100}{50} = 2$$

Let t be the lowest temperature to which oil may be cooled and this will be the highest temperature of water too.

Hence, $$\dot{m}_h\, c_{ph}(200-t) = \dot{m}_c\, c_{pc}(t-20) \Rightarrow (200-t) = \frac{\dot{m}_c\, c_{pc}}{\dot{m}_h\, c_{ph}}(t-20) = 2(t-20)$$

Or, 200-t=2t-40

Or, $t=80^0C$ (Ans.)

Q. A chemical having specific heat of 3.5 kJ/kg K flowing at the rate of $18X10^3$ kg/hr enters a parallel flow heat exchanger at 110^0C. The flow rate of cooling water is $48X10^3$ kg/hr with an inlet temperature of 27^0C . The heat transfer area is $12m^2$ and the overall heat transfer coefficient is 1030 W/m^2K. Find out

- **(i) Effectiveness of heat exchanger,**
- **(ii) Outlet temperature of water,**
- **(iii) Outlet temperature of chemical.**

Solution:

Given data:

C_{ph} =3.5kJ/kgK , m_h =18000kg/hr=18000/3600=5kg/s

t_{h1} =110^0C, m_c =48000kg/hr=48000/3600=13.3kg/s

c_{ph} =4.186 kJ/kg K, $t_{c1}=27^0C$

$A=12m^2$, $U=1030\ W/m^2K$

Hot fluid(chemical) capacity rate

$C_h = m_h c_{ph} = 5X3.5 = 17.5$

Cold fluid capacity rate

$C_c = m_c c_{pc} = 13.3X4.186 = 55.67$

Hence $C_h < C_c$

Heat lost by hot fluid=Heat gained by cold fluid

$$\Rightarrow 5X3.5(110 - t_{h2}) = 13.3X4.186(t_{c2} - 27) \Rightarrow (110 - t_{h2}) = 3.18(t_{c2} - 27)................(1)$$

Now $NTU = \dfrac{UA}{C_{\min}} = \dfrac{1030X12}{17.5X1000} = 0.706$

$$\varepsilon = \frac{1 - \exp\left[- NTU(1 + R)\right]}{1 + R}$$

$$R = \frac{C_{\min}}{C_{\max}} = \frac{17.5}{55.67} = 0.314$$

$$\Rightarrow \varepsilon = \frac{1 - \exp\left[-0.706(1 + 0.314)\right]}{1 + 0.314} = 0.46$$

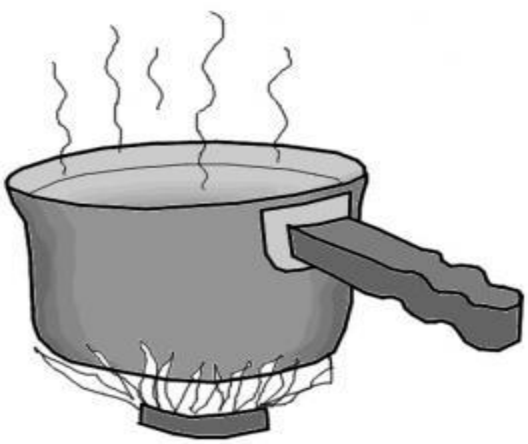
Outlet temperature of water (t_{c2})

We know that $\varepsilon = \dfrac{C_h(t_{h1} - t_{h2})}{C_{\min}(t_{h1} - t_{c1})} \Rightarrow 0.46 = \dfrac{110 - t_{h2}}{110 - 27} \Rightarrow t_{h2} = 71.82^0 C$

Substituting the value of t_{h2} in equation (1)

$$(110 - 71.82) = 3.18(t_{c2} - 27) \Rightarrow t_{c2} = 39^0 C$$

BOILING AND CONDENSATION



Q. A wire of 1.2 mm diameter and 200mm length is submerged horizontally in water at 7 bar. The wire carries a current of 135 A with an applied voltage of 2.18 V. If the surface of the wire is maintained at 200^{0}C, calculate

i. **The heat flux and**

ii. **The boiling heat transfer coefficient.**

Solution:

Given: d=1.2 mm=0.0012 m. l=200 mm= 0.2m, I =135 A, V=2.18 V, t_s=200^{0}C.

(i) The heat flux,q:

The electrical energy input to the wire is given by

Q=VI=2.18X135=294.3 W

Surface area of the wire, A=Πdl=ΠX0.0012X0.2=7.54X10^{-4} m^2

Hence q=Q/A=294.3/7.54X10^{-4}=0.39X10^6 W/m^2 =0.39 MW/m^2(Ans.)

(ii) The boiling heat transfer coefficient, h"

Corresponding to 7 bar, t_{sat}=164.97^{0}C and q=h(t_s-t_{sat})

H=q/(t_s-t_{sat})=0.39X10^6/(200-164.97)=11133.3W/m^{20}C. (Ans.)

Q. An electric wire of 1.25 mm diameter and 250 mm long is laid horizontally and submerged in water at atmospheric pressure. The wire has an applied voltage of 18 V and carries a current of 45 amperes. Calculate :

(i) The heat flux, and

(ii) The excess temperature.

The following correlation for water boiling on horizontal submerged surface holds good:

$$h = 1.58\left(\frac{Q}{A}\right)^{0.75} = 5.62(\Delta t_e)^3, W/m^{20}C$$

Solution:

Given, d=1.25 mm=0.00125 m , l=250 mm=0.25m, V=18 V, I=45 A

(i) The heat flux,q:

Electrical energy input to the wire, Q=VI=18X45=810 W

Surface area of the wire, A_s=Πdl=ΠX0.00125X0.25=9.817X10^{-4}m^2

Hence, q=Q/A=810/(9.817X10^{-4})=0.825X10^{6}W/m^2=0.825 MW/m^2(Ans.)

(ii) The excess temperature. Δt_e:

Using the correlation,

$$h = 1.58\left(\frac{Q}{A}\right)^{0.75} = 5.62(\Delta t_e)^3 \Rightarrow 1.58(0.825X10^6)^{0.75} = 5.62(\Delta t_e)^3$$

$$\Rightarrow \Delta t_e = \left[\frac{1.58(0.825X10^6)^{0.75}}{5.62}\right]^{0.333} = 19.68^0 C(Ans.)$$

Q. A nickel wire of 1mm diameter and 400 mm long, carrying current, is submerged in a water bath which is open to atmospheric pressure . Calculate the voltage at the burnout point if at this point the wire carries a current of 190 A.

Solution:

Given:

D= 1 mm =0.001 m, l=400 mm=0.4 m, I=190 A

The thermo-physical properties of water and vapour at 100^0C are:

$P_l=\rho_f$=958.4 kg/m^3, ρ_v=0.5955 kg/m^3, h_{fg}=2257kJ/kg, σ=58.9X 10^{-3} N/m.

Voltage at the burnout point, V_b:

At burnout point critical heat flux is

$$q_{sc} = 0.18(\rho_v)^{1/2} h_{fg}[g\sigma(\rho_l - \rho_v)]^{1/4} = 0.18(0.5955)^{1/2} X\, 2257 X 10^3 [9.81 X 58.9 X 10^{-3}(958.4 - 0.5955)]^{1/4}$$

=1.52X10^6 W/m^2=1.52MW/m^2

Electric energy input to the wire,

Q=V_b X I

Or, q=Q/A=(V_b X I)/A=q_{sc}

Or, V_b =(AXq_{sc})/I=ΠdlXq_{sc}/I=10.05 V. (Ans.)